**APPENDIX 2: Mathematical Appendix for Spending Change Calculations**

Suppose we assign variables as follows, for Period 1 (before a change in the local tax rate) and Period 2 (after a change in the local tax rate):

$$A_i = Allocation\ in\ each\ Period$$
$$Ei \quad = Taxable\ expenditures\ in\ each\ Period$$
$$S_i = Total\ spending\ in\ each\ Period$$
$$T_i = Tax\ rate\ in\ each\ Period$$
$$\beta = Regression\ coefficient\ from\ Table\ 4$$
$$\Delta_T = Percent\ change\ in\ the\ tax\ rate\ from\ Period\ 1\ to\ Period\ 2$$
$$\Delta_A = Percent\ change\ in\ Arlington\ tax\ revenues\ from\ Period\ 1\ to\ Period\ 2$$
$$\Delta_E = Percent\ change\ in\ taxable\ expenditures\ from\ Period\ 1\ to\ Period\ 2$$
$$\Delta_S = Percent\ change\ in\ total\ spending\ from\ Period\ 1\ to\ Period\ 2$$

$$i = 1, 2$$

We can begin by defining $A_1,\ E_1,\ A_2,\ E_2,\ S_1,\ and\ S_2$ simply as:

$A_1 = T_1 E_1\ and\ A_2 = T_2 E_2$,

$$E_1 = \frac{A_1}{T_1}\ and\ E_2 = \frac{A_2}{T_2},$$

$S_1 = (1 + T_1)E_1\ and\ S_2 = (1 + T_2)E_2.$

Additionally, we define $\Delta_T,\ \Delta_A,\ \Delta_E\ , \Delta_S$ as:

$\Delta_T = \frac{T_2}{T_1} - 1, \Delta_A = \frac{A_2}{A_1} - 1, \Delta_E = \frac{E_2}{E_1} - 1, and\ \Delta_S = \frac{S_2}{S_1} - 1.$

Then it follows naturally from the form of the regression model that our percentage change for the tax increment in question will be $\Delta_A = (\Delta_T + 1)^\beta - 1 = \left(\frac{T_2}{T_1}\right)^\beta - 1$.

From here, we can substitute in order to calculate the analogous percent changes in both taxable expenditures and total spending in Periods 1 and 2, beginning with $A_2$:

$A_2 = (1 + \Delta_A)A_1 = \left(\frac{T_2}{T_1}\right)^\beta T_1 E_1,$

by combining the fact that $A_1 = T_1 E_1$ and $\Delta_A = \left(\frac{T_2}{T_1}\right)^\beta - 1$. from above.

Now, we can represent $\Delta_E$ in terms of only our elasticity from the regression model and the analogous tax rates in the two periods, beginning with substitution into $E_2$ from above:

$$E_2 = \frac{A_2}{T_2} = \frac{\left(\frac{T_2}{T_1}\right)^{\beta} T_1 E_1}{T_2},$$

Substituting this into the definition for $\Delta_E$,

$$\Delta_E = \frac{E_2}{E_1} - 1 = \frac{\left(\frac{T_2}{T_1}\right)^{\beta} T_1 E_1}{T_2 E_1} - 1,$$

and cancelling $E_1$ from the top and bottom,

$$\Delta_E = \frac{\left(\frac{T_2}{T_1}\right)^{\beta} T_1}{T_2} - 1 = \left(\frac{T_1}{T_2}\right)\left(\frac{T_2}{T_1}\right)^{\beta} - 1$$

$$= \left(\frac{T_1}{T_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{T_2}{T_1}\right)^{\beta-1} - 1$$

$$= \left(\frac{T_2}{T_1}\right)^{\beta-1} - 1.$$

Lastly, we can evaluate a simple expression of known values for the change in total spending in the area across the two periods:

$$\Delta_S = \frac{(1+T_2)E_2}{(1+T_1)E_1} - 1, by\ substitution\ from\ S_i = (1+T_i)E_i,$$

$$\frac{E_2}{E_1} = \Delta_E + 1, by\ rearrangement\ from\ the\ definition\ \Delta_E = \frac{E_2}{E_1} - 1,$$

$$\Delta_S = \frac{(1+T_2)}{(1+T_1)}(\Delta_E + 1) - 1, by\ substitution.$$

We simplify this to:

$$\Delta_S = \frac{(1+T_2)}{(1+T_1)}\left[\left(\left(\frac{T_2}{T_1}\right)^{\beta-1} - 1\right) + 1\right] - 1$$

$$= \frac{(1+T_2)}{(1+T_1)}\left(\frac{T_2}{T_1}\right)^{\beta-1} - 1$$

Once the regression model is estimated, the elasticity for the local tax rate can be substituted for β, and the known tax rates in the two periods are substituted for $T_1$ *and* $T_2$. For the change from a 1.0% to a 1.5% local tax rate:

$\beta = 0.893,$
$T_1 = 0.010$
$T_2 = 0.015$

$$\Delta_{\boldsymbol{A}} = \left(\frac{T_2}{T_1}\right)^{\beta} - 1 = \left(\frac{0.015}{0.010}\right)^{0.893} - 1 \approx 0.4363 = \mathbf{43.63}\%,$$

$$\Delta_{\boldsymbol{E}} = \left(\frac{T_2}{T_1}\right)^{\beta-1} - 1 = \left(\frac{0.015}{0.010}\right)^{0.893-1} - 1 \approx -0.0425 = \mathbf{-4.25}\%,$$

$$\Delta_{\boldsymbol{S}} = \frac{(1+T_2)}{(1+T_1)}\left(\frac{T_2}{T_1}\right)^{\beta-1} - 1 = \frac{(1+0.015)}{(1+0.010)}\left(\frac{0.015}{0.010}\right)^{0.893-1} - 1 \approx -0.0377 = \mathbf{-3.77}\%.$$

Similarly, for the change in the local tax rate from 1.25% to 1.75%:

$\beta = 0.893,$
$T_1 = 0.0125$
$T_2 = 0.0175$

$$\Delta_{\boldsymbol{A}} = \left(\frac{T_2}{T_1}\right)^{\beta} - 1 = \left(\frac{0.0175}{0.0125}\right)^{0.893} - 1 \approx 0.3505 = \mathbf{35.05}\%,$$

$$\Delta_{\boldsymbol{E}} = \left(\frac{T_2}{T_1}\right)^{\beta-1} - 1 = \left(\frac{0.0175}{0.0125}\right)^{0.893-1} - 1 \approx -0.0354 = \mathbf{-3.54}\%,$$

$$\Delta_{\boldsymbol{S}} = \frac{(1+T_2)}{(1+T_1)}\left(\frac{T_2}{T_1}\right)^{\beta-1} - 1 = \frac{(1+0.0175)}{(1+0.0125)}\left(\frac{0.0175}{0.0125}\right)^{0.893-1} - 1 \approx -0.0306 = \mathbf{-3.06}\%.$$